STL^3

Toward Security via Free Theorems*
in a **S**ession-**T**yped **L**inear **L**anguage with **L**ocations

*Work in Progress!

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February 1, 2024

Northeastern University

Secure Coin Flipping

COIN FLIPPING BY TELEPHONE
A PROTOCOL FOR SOLVING IMPOSSIBLE PROBLEMS

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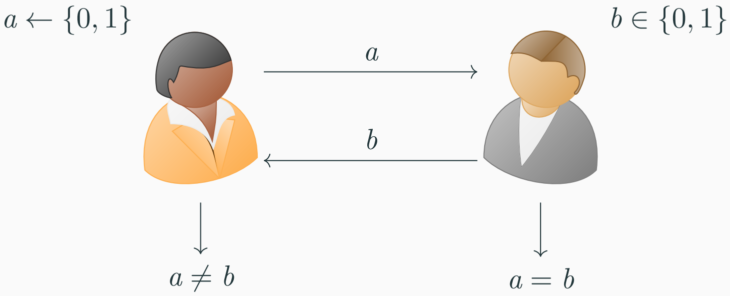
November 10, 1981

Abstract

Alice and Bob want to flip a coin by telephone. (They are very good friends , live in different cities, want to decide who will travel to see whom.) Bob would not like to tell Alice HEADS and hear Alice (at the other end of the line) say "Here goes... I'm flipping the coin.... You lost!"

Insecure Coin Flipping



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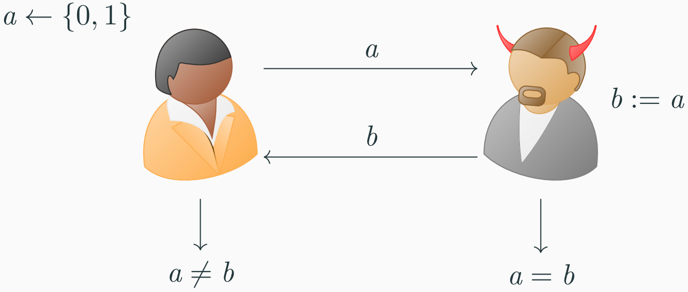

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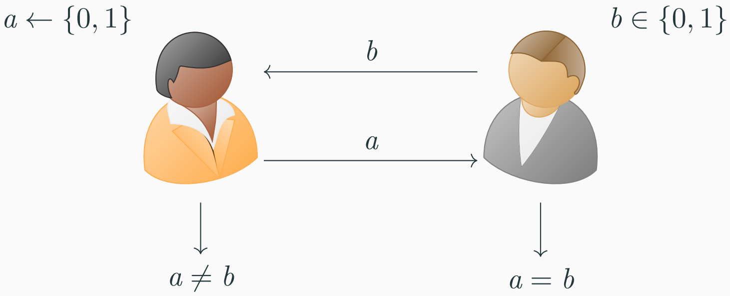

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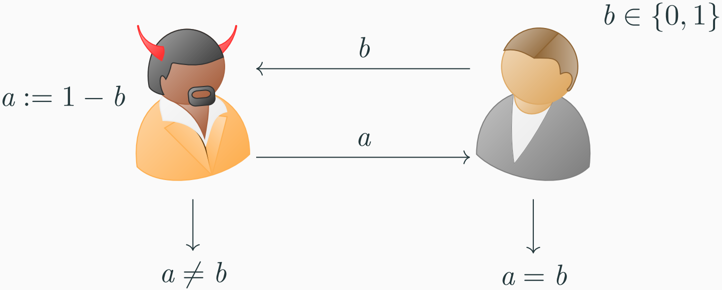
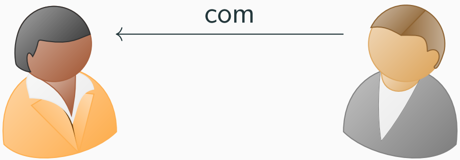


Secure Coin Flipping

$$b \in \{0, 1\}$$

$$(\mathsf{com}, \mathsf{dec}) \leftarrow \mathsf{Commit}(b)$$



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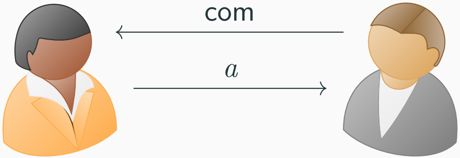

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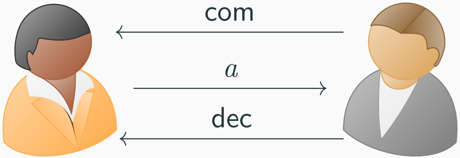
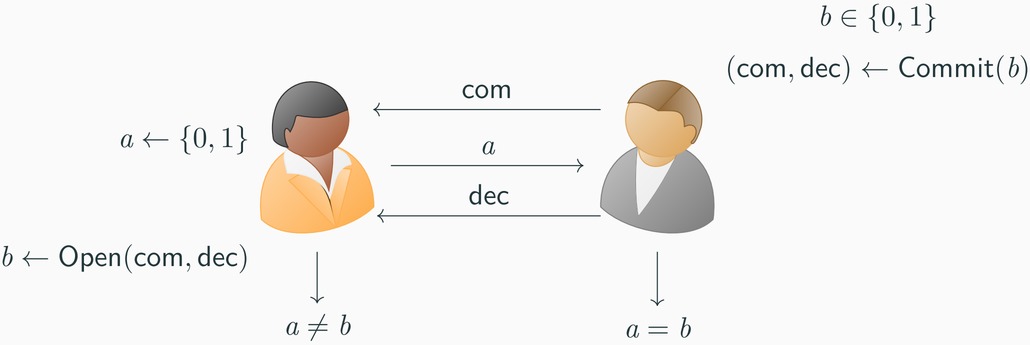


$b \leftarrow \mathsf{Open}(\mathsf{com}, \mathsf{dec})$

$a \neq b$

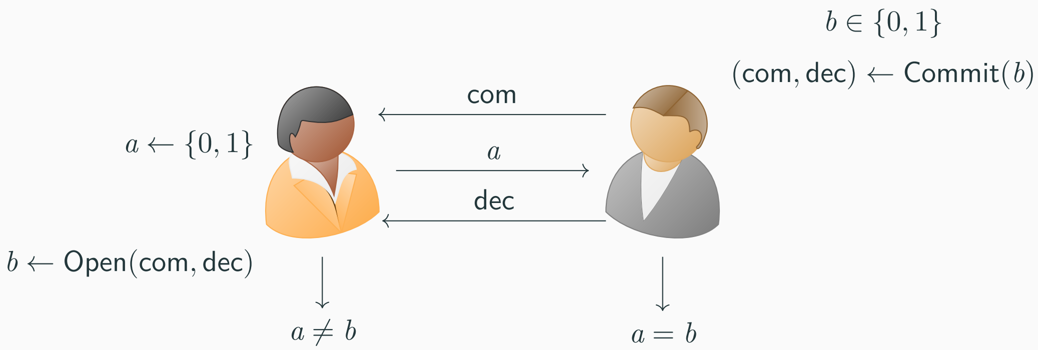
$a = b$

Secure Coin Flipping



Can we express this protocol with session types? Definitely!

Secure Coin Flipping



Can we express the security of this protocol with session types?

Key Ideas

- $\star$ Types can reference *particular* processes by name: $@_x\, T$

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- ... and can *quantify* over names, establishing name parametricity: $\forall\, x.\, T, \exists\, x.\, T$

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- Types can reference *particular* processes by name: $@_x\, T$
- … and can *quantify* over names, establishing name parametricity: $\forall\, x. T, \exists\, x. T$
- Security[1] of some protocols can be expressed as free theorems

[1] information flow

Background: Intuitionistic Binary Sessions[2]

$\boxed{\Delta \vdash \mathcal{P} :: (x : T)}$ Δ a lin. ctx. of channels; $\mathcal{P}$ a process; x a name; T a type

[2] Caires and Pfenning 2010.

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$\multimap R$

$$\frac{\Delta, y : S \vdash \mathcal{P} :: (x : T)}{\Delta \vdash y \leftarrow x.\textbf{recv}; \mathcal{P} :: (x : S \multimap T)}$$

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$\multimap L^*$

$$\frac{\Delta, y : S_2 \vdash \mathcal{P} :: (z : T)}{\Delta, x : S_1, y : S_1 \multimap S_2 \vdash y.\textbf{send}(x); \mathcal{P} :: (z : T)}$$

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$\multimap / \otimes$
$$\langle \hat{x}.\textbf{send}(\hat{y}); \mathcal{P} \rangle \quad \langle y \leftarrow \hat{x}.\textbf{recv}; \mathcal{Q} \rangle \to \langle \mathcal{P} \rangle \quad \langle \mathcal{Q}[\hat{y}/y] \rangle$$

[2] Caires and Pfenning 2010.

Background[3]: L^3

$$\boxed{\mathsf{ref}\ T \cong \exists\,\ell.\ !\mathsf{ptr}\,\ell \otimes \mathsf{cap}\,\ell\ T}$$

Decompose a traditional *reference* into an unrestricted *pointer* and a linear *capability*, tied together by a type-level name, ℓ.

[3] Ahmed, Fluet, and Morrisett 2007.

STL^3 Extensions: Exchanging Names

$\boxed{\Gamma; \Delta \vdash \mathcal{P} :: (x : T)}$ Γ an unrestricted context of names, $\Gamma \supseteq$ freenames(Δ, T)

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$$\text{cut} \quad \frac{\Gamma; \Delta_1 \vdash \mathcal{P} :: (x : S) \qquad \Gamma, x; \Delta_2, x : S \vdash \mathcal{Q} :: (y : T)}{\Gamma; \Delta_1, \Delta_2 \vdash x \leftarrow \langle \mathcal{P} \rangle ;\ \mathcal{Q} :: (y : T)}$$

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$$\forall R \quad \frac{\Gamma, y; \Delta \mathcal{P} :: (x : T)}{\Gamma; \Delta \vdash [y] \leftarrow x.\textbf{recv}; \mathcal{P} :: (x : \forall\, y. T)}$$

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$$\forall L \quad \frac{\Gamma; \Delta, y : S[x/x'] \vdash \mathcal{P} :: (z : T) \qquad \Gamma \ni x}{\Gamma; \Delta, y : \forall\, x'.S \vdash y.\textbf{send}[x]; \mathcal{P} :: (z : T)}$$

STL[3] Extensions: Jumping[4]

$\boxed{\Gamma; \Delta \vdash \mathcal{P} :: (x : T)}$ Γ an unrestricted context of names, $\Gamma \supseteq \text{freenames}(\Delta, T)$

$@_x R$

$\Gamma; x : T \vdash y.\textbf{send}(@x) :: (y : @_x T)$

[4] Braüner and Paiva 2006.

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$$\dfrac{}{\Gamma; x : T \vdash y.\mathbf{send}(@x) :: (y : @_x\, T)}\ @_x R \qquad \dfrac{\Gamma; x : S \vdash \mathcal{P} :: (z : T)}{\Gamma; y : @_x\, S \vdash @x \leftarrow y.\mathbf{recv}; \mathcal{P} :: (z : T)}\ @_x L$$

$\boxed{\delta \to \delta'}$

$$\dfrac{}{\langle \hat{y}.\mathbf{send}(@\hat{x})\rangle \ \ \langle @\hat{x} \leftarrow \hat{y}.\mathbf{recv}; \mathcal{P}\rangle \to \langle \mathcal{P}\rangle}\ @$$

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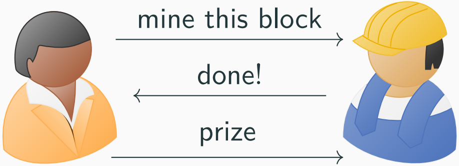
Example: Proof of Work



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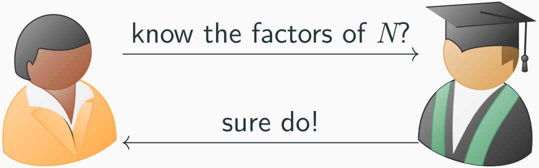
$$\boxed{\mathsf{PoW} \triangleq \forall\, i.\ \underbrace{@_i\, T}_{\text{task}} \multimap \underbrace{@_i\, \mathbb{1}}_{\text{cert}} \otimes \underbrace{\mathbb{N}}_{\text{prize}} \multimap \mathbb{1}}$$

Example: Zero-Knowledge Proof



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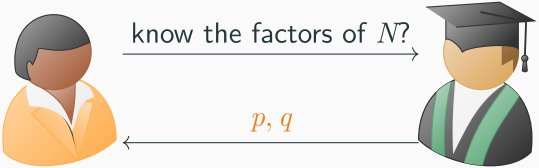

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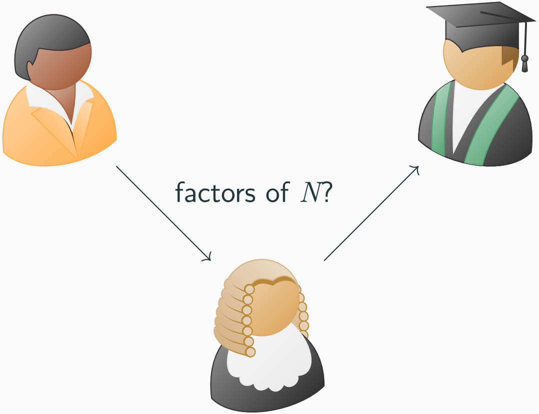

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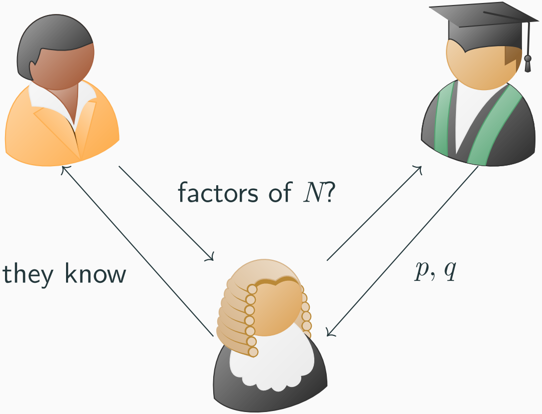

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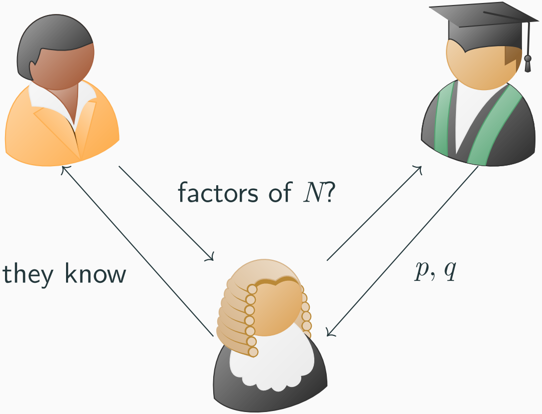

$$\boxed{\mathsf{ZKP} \triangleq \forall\, i.\ \underbrace{@_i\,(\mathbb{N}^2 \multimap \mathbb{B})}_{\text{test}} \multimap \underbrace{@_i\,\mathbb{B}}_{\text{cert}}}$$

Security Property: Authenticity

$$\boxed{\mathsf{Adv} \triangleq \forall\, e.\ \underbrace{@_e \mathbb{B}}_{\text{sign}} \multimap \underbrace{@_e \mathbb{B}}_{\text{forge?}}}$$

“Can an adversary forge a named process?”

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"Can an adversary forge a named process?"

Conjecture (Authenticity)

For all $\mathcal{E} : \mathbb{B}$, $\mathcal{A} : \mathsf{Adv}$,

$$\mathcal{E} \approx e \leftarrow \mathcal{E}; \mathsf{adv} \leftarrow \mathcal{A}; \mathsf{exp}(\mathsf{adv}, e)$$

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"Can an adversary forge a named process?"

$$
\begin{array}{l}
\textbf{proc}\ \mathsf{exp}(\mathsf{adv} : \mathsf{Adv}, e : \mathbb{B}) : \mathbb{B} \triangleq \\
\quad \mathsf{adv}.\textbf{send}[e]; \\
\quad \mathsf{sig} \leftarrow \langle \mathsf{sig.send}(@e) \rangle\ ; \\
\quad \mathsf{adv}.\textbf{send}(\mathsf{sig}); \\
\quad @e \leftarrow \mathsf{adv}.\textbf{recv}; \\
\quad \mathsf{exp}.\textbf{fwd}(e)
\end{array}
$$

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Security Property: Hiding

$$\boxed{\mathsf{Adv} \triangleq \forall\, e.\ \underbrace{@_e\, \mathbb{B}}_{\text{enc}} \multimap \underbrace{@_e\, \mathbb{B}}_{\text{ack}} \otimes \underbrace{\mathbb{B}}_{\text{guess?}}}$$

"Can an adversary peek into a named process?"

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$\quad \mathsf{adv}.\textbf{send}(\mathsf{enc});$
$\quad \mathsf{enc} \leftarrow \mathsf{adv}.\textbf{recv};$
$\quad @e \leftarrow \mathsf{enc}.\textbf{recv};$
$\quad e.\textbf{recv}\{\overline{\cdot \Rightarrow \mathsf{exp}.\textbf{fwd}(\mathsf{adv})}\}$

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Security Property: Binding

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"Can an adv. switch a named process?"

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"Can an adv. switch a named process?"

$\textbf{proc}\ \mathsf{exp}(\mathsf{adv} : \mathsf{Adv}, e : \mathbb{B}) : \mathbb{B} \triangleq$

$\quad [a] \leftarrow \mathsf{adv}.\textbf{recv};$

$\quad \mathsf{com} \leftarrow \mathsf{adv}.\textbf{recv};$

$\quad \mathsf{adv}.\textbf{send}(\mathsf{com});$

$\quad \mathsf{adv}.\textbf{send}(e);$

$\quad @a \leftarrow \mathsf{adv}.\textbf{recv};$

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Conjecture (Binding)

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Secure Coin Flipping in STL[3]

$$\mathsf{Flip} \triangleq \forall\, c.\ \underbrace{@_c\,\mathbb{B}}_{\mathsf{com}} \multimap \underbrace{@_c\,\mathbb{B}}_{\mathsf{ack}} \otimes \underbrace{\mathbb{B}}_{\mathsf{flip}} \otimes \underbrace{@_c\,\mathbb{B}}_{\mathsf{opn}} \multimap \mathbb{1}$$

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proc flip() : Flip $\triangleq$

  $[c]$(com) $\leftarrow$ flip.**recv**;

  flip.**send**(com);

  flip.**send** $\langle f \leftarrow$ sample()$\rangle$;

  com $\leftarrow$ flip.**recv**;

  $@c \leftarrow$ com.**recv**;

  $\ldots$ *use the result,* c

  flip.**close**

Secure Coin Flipping in STL[3]

$\text{Flip} \triangleq \forall\, c.\ \underbrace{@_c\,\mathbb{B}}_{\text{com}} \multimap \underbrace{@_c\,\mathbb{B}}_{\text{ack}} \otimes \underbrace{\mathbb{B}}_{\text{flip}} \otimes \underbrace{@_c\,\mathbb{B}}_{\text{opn}} \multimap \mathbb{1}$ $\quad \text{Call} \triangleq \exists\, c.\ \underbrace{@_c\,\mathbb{B}}_{\text{com}} \otimes \underbrace{@_c\,\mathbb{B}}_{\text{ack}} \multimap \underbrace{\mathbb{B}}_{\text{flip}} \multimap \underbrace{@_c\,\mathbb{B}}_{\text{opn}}$

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$\textbf{proc}\ \mathsf{flip}()\ :\ \mathsf{Flip} \triangleq$

- $[c](\mathsf{com}) \leftarrow \mathsf{flip}.\textbf{recv};$
- $\mathsf{flip}.\textbf{send}(\mathsf{com});$
- $\mathsf{flip}.\textbf{send}\,\langle f \leftarrow \mathsf{sample}()\rangle\,;$
- $\mathsf{com} \leftarrow \mathsf{flip}.\textbf{recv};$
- $@c \leftarrow \mathsf{com}.\textbf{recv};$
- $\ldots$ *use the result,* c
- $\mathsf{flip}.\textbf{close}$

$$\mathsf{Call} \triangleq \exists\, c.\ \underbrace{@_c\,\mathbb{B}}_{\mathsf{com}} \otimes \underbrace{@_c\,\mathbb{B}}_{\mathsf{ack}} \multimap \underbrace{\mathbb{B}}_{\mathsf{flip}} \multimap \underbrace{@_c\,\mathbb{B}}_{\mathsf{opn}}$$

$\textbf{proc}\ \mathsf{call}()\ :\ \mathsf{Call} \triangleq$

- $c \leftarrow \mathsf{guess}();$
- $\mathsf{call}.\textbf{send}[c]\,\langle \mathsf{com} \leftarrow \mathsf{com}.\textbf{send}(@c)\rangle\,;$
- $\mathsf{com} \leftarrow \mathsf{call}.\textbf{recv};$
- $f \leftarrow \mathsf{call}.\textbf{recv};$
- $\ldots$ *use the result,* f
- $\mathsf{call}.\textbf{fwd}(\mathsf{com})$

Coin Flipping: Flipper Security

$$\text{Call} \triangleq \exists\, c.\ @_c\mathbb{B} \otimes @_c\mathbb{B} \multimap \mathbb{B} \multimap @_c\mathbb{B}$$

$$
\begin{aligned}
&\textbf{proc}\ \text{exp}(\text{call} : \text{Call}, b : \mathbb{B}) : \mathbb{B} \triangleq \\
&\quad [c](\text{com}) \leftarrow \text{call}.\textbf{recv}; \\
&\quad \text{call}.\textbf{send}(\text{com}); \\
&\quad \text{call}.\textbf{send}(b); \\
&\quad \text{com} \leftarrow \text{call}.\textbf{recv}; \\
&\quad @c \leftarrow \text{com}.\textbf{recv}; \\
&\quad \text{exp}.\textbf{fwd}(c)
\end{aligned}
$$

Conjecture (Flipper Security)

Forall $\mathcal{C}$ :: (call : Call),

$$\text{call} \leftarrow \mathcal{C}; \text{exp}(\text{call}, 0) \approx \text{call} \leftarrow \mathcal{C}; \text{exp}(\text{call}, 1)$$

Coin Flipping: Caller Security

$$\text{Flip} \triangleq \forall\, c.\ @_c\,\mathbb{B} \multimap @_c\,\mathbb{B} \otimes \mathbb{B} \otimes @_c\,\mathbb{B} \multimap \mathbb{1}$$

$$
\begin{array}{l}
\textbf{proc}\ \text{exp}(\text{flip} : \text{Flip}, c : \mathbb{B}) : \mathbb{B} \triangleq \\
\quad \text{flip}.\textbf{send}[c]; \\
\quad \text{com} \leftarrow \langle \text{com}.\textbf{send}(@c) \rangle\ ; \\
\quad \text{flip}.\textbf{send}(\text{com}); \\
\quad \text{com} \leftarrow \text{flip}.\textbf{recv}; \\
\quad f \leftarrow \text{flip}.\textbf{recv}; \\
\quad \text{flip}.\textbf{send}(\text{com}); \\
\quad \text{flip}.\textbf{wait}; \\
\quad \text{exp}.\textbf{fwd}(f)
\end{array}
$$

Conjecture (Caller Security)

Forall $\mathcal{F}$ :: (flip : Flip),

$$\text{flip} \leftarrow \mathcal{F}; \text{exp}(\text{flip}, 0) \approx \text{flip} \leftarrow \mathcal{F}; \text{exp}(\text{flip}, 1)$$

But... we can’t prove it yet :)

Relationship to Type Polymorphism

No Reveal $\forall\, i.\ @_i\,\mathbb{B}^n \multimap @_i\,\mathbb{B}^n \quad \rightsquigarrow \quad \forall\, \alpha.\ \alpha \multimap \alpha$

Relationship to Type Polymorphism

No Reveal	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap @_i\,\mathbb{B}^n$	$\rightsquigarrow$	$\forall\, \alpha.\ \alpha \multimap \alpha$
Partial Reveal	$\forall\, i.\ @_i\,\mathbb{B}^{m+n} \multimap @_i\,\mathbb{B}^n$	$\rightsquigarrow$	$\forall\, \alpha.\ \mathbb{B}^m[\alpha] \multimap \alpha$

Relationship to Type Polymorphism

No Reveal	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap @_i\,\mathbb{B}^n$	$\rightsquigarrow$	$\forall\, \alpha.\ \alpha \multimap \alpha$
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Cond. Reveal	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap \mathbb{1} \mathbin{\&} @_i\,\mathbb{B}^n$	$\not\rightsquigarrow$	$\forall\, \alpha.\ \alpha \multimap \big(\underbrace{(\alpha \multimap \mathbb{B})}_{\text{not binding!}} \multimap \mathbb{1}\big) \mathbin{\&} \alpha$

Relationship to Type Polymorphism

No Reveal	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap @_i\,\mathbb{B}^n$	$\rightsquigarrow$	$\forall\, \alpha.\ \alpha \multimap \alpha$
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Cond. Reveal	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap \mathbb{1}\ \&\ @_i\,\mathbb{B}^n$	$\not\rightsquigarrow$	$\forall\, \alpha.\ \alpha \multimap \Big(\underbrace{(\alpha \multimap \mathbb{B})}_{\text{not binding!}} \multimap \mathbb{1}\Big)\ \&\ \alpha$
Type-Sensitive	$\forall\, i.\ @_i\,(\mathbb{N} \multimap \mathbb{B}) \multimap @_i\,\mathbb{B}$	$\not\rightsquigarrow$	$\forall\, \alpha.\ \underbrace{(\mathbb{N} \multimap \alpha)}_{\text{not ZK!}} \multimap \alpha$

Logical Relations[5]

$$
\boxed{\begin{array}{lclll}
(v_1, v_2) & \in & \mathcal{V}[\![\forall\,\alpha.\,T]\!]\rho & \text{iff} & \text{for all types } S_1, S_2 \text{ and candidates } R, \\
 & & & & (v_1[S_1], v_2[S_2]) \in \mathcal{E}[\![\,T]\!]\rho[\alpha \mapsto R] \\
(v_1, v_2) & \in & \mathcal{V}[\![\alpha]\!]\rho & \text{iff} & (v_1, v_2) \in \rho(\alpha)
\end{array}}
$$

Conditional revealing means candidacy must be more strict: it is constrained by how the name is used in the continuation type.

[5] Reynolds 1983.

Candidates?

No Restriction $\forall\, i.\ @_i\, \mathbb{B}^n \multimap @_i\, \mathbb{B}^n$

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No Restriction	$\forall\, i.\ @_i\, \mathbb{B}^n \multimap @_i\, \mathbb{B}^n$
Equivalent	$\forall\, i.\ @_i\, \mathbb{B}^n \multimap \mathbb{1}$

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No Restriction	$\forall\, i.\ @_i\, \mathbb{B}^n \multimap @_i\, \mathbb{B}^n$
Equivalent	$\forall\, i.\ @_i\, \mathbb{B}^n \multimap \mathbb{1}$
	$\forall\, i.\ @_i\, \mathbb{B}^n \multimap \mathbb{1} \oplus @_i\, \mathbb{B}^n$

Candidates?

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	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap \mathbb{1} \oplus @_i\,\mathbb{B}^n$
$\frac{1}{2}$ **Equivalent**	$\forall\, i.\ @_i\,\mathbb{B}^{2n} \multimap @_i\,\mathbb{B}^n$

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	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap \mathbb{1} \oplus @_i\,\mathbb{B}^n$
$\frac{1}{2}$ **Equivalent**	$\forall\, i.\ @_i\,\mathbb{B}^{2n} \multimap @_i\,\mathbb{B}^n$
Conditionally Equivalent	$\forall\, i.\ @_i\,\mathbb{B}^n \multimap \mathbb{1}\ \&\ @_i\,\mathbb{B}^n$

Other Proof Ideas

- Computation focusing (Rioux and Zdancewic 2020)
- Theorems for free from separation logic specifications (Birkedal et al. 2021)
- Session logical relations for noninterference (Derakhshan, Balzer, and Jia 2021)
- Suggestions?